Name:_________________________ Period:_____________

# Relationship between Speed and Height of Roller Coasters

In the previous activity, you investigated how the height of a paragraph changes with its font size. You modeled the non-linear relationship and interpreted the parameter in context of the data. In this activity, you will work with data on roller coasters.

Your task is to fit the data and find the function that describes the relationship.

## Introduction

The data in this activity summarize the height and speed of 20 roller coasters in the United States.

In this activity, you will investigate: *How does the speed of a rollercoaster depend on its height?*

To this end, you will fit the data and to find the function that describes the relationship

- Open the Tuva dataset at: tuva.la/2L2BWQT
- Click on "About" to learn more about the data

**★*Tuva Tip: Roller Coaster Science You Should Know:*★**

*A roller coaster at the top of a slope has what's called* ***potential energy (PE)****, and as it goes down—and speeds up—that potential energy gets converted into* ***kinetic energy (KE)****.*

*Potential energy is proportional to how high you are. Its formula is* **$PE = mgh$***, where m is your mass, h is the height, and g is the acceleration of gravity.*

*Kinetic energy depends on your speed. Its formula is* **$KE = \frac{1}{2} mv^2$***, where m is mass and v is speed.*

***Law of Conservation of Energy:***

*If no energy is lost as the roller coaster goes down the slope, all of the potential energy gets converted to kinetic, that is,*

**$PE = KE$ or, $mgh = \frac{1}{2} mv^2$**

## **1) Investigate**: How does the speed of a roller coaster depend on its height?

**Q1:** Which is the independent variable?

____ a. Speed
____ b. Height

**Q2:** Why is it the independent variable?

____ a. Height causes speed, so height is the independent variable
____ b. The higher the speed, the greater the height, thus speed is the independent variable

**Now Graph it:**

- Drag **Height** to the x-axis.
- Drag **Speed** to the y-axis.

**Use the Law of Conservation of Energy:**

We will now use the law of conservation of energy to write the function that models the relationship between height and speed.

★ *Tuva Tip: Hint* ★

- ➔ Remember that you know: **PE=KE or, $mgh = \frac{1}{2} mv^2$**
- ➔ $v$ in the equation stands for speed (y-variable), while h stands for height (x-variable).

**Q3:** Which of the following equations will you get after solving for speed ($v$)?

____ a. $v = 2gh$
____ b. $v = \frac{1}{2} gh^2$
____ c. $v = \sqrt{2gh}$

★ *Tuva Tip: Hint* ★

- ➔ We need a square root function to approximate the relationship.
- ➔ Thus, Speed = 2*g**height, where *g* is the parameter.
- ➔ Remember, the parameter is a specific number we will find when we fit the function to our data.

## Build Your Function

- Choose f(x) from the toolbar above the graphing area to open the Modeling Card
- Click on the down arrow on the function editor to open the drop-down menu.
- Choose a*√x (the generic form of the square root function) from the menu.
- You don't need the parameter a in front of the radical, so delete it.
- Edit the the function under the square root sign to input (y = $\sqrt{2g * x}$)).
- Notice that this is the same as: Speed = $\sqrt{2g * height}$ with symbols for speed and height.

Two things should happen:

1. Your function appears on the graph.
2. The parameter ***g*** appears as a slider below the function.

## Explore the parameter

- Click the "-" sign on the top left to zoom out.

★ *Tuva Tip: Tweaking the Parameter in Tuva* ★

- ➔ You can change the value of a parameter by dragging the slider pointer or by typing in a new value and pressing enter. As you do this, the curve will move.
- ➔ You can also tweak the maximum value of ***g*** on the slider by clicking on it and inputting the desired value in the box in the middle.
- ➔ Additionally, if you feel that you need to make very small changes to the parameter value, double click on the upper range of the slider and adjust the steps to a smaller value such as 0.01.

**Q4:** How does the function behave when you make ***g*** larger than 1?

____ a. It gets stretched vertically
____ b. It gets compressed horizontally
____ c. It shifts horizontally
____ d. It gets reflected along the x-axis

**Q5:** How does the function behave when you make ***g*** smaller than 1, but keep it positive (between 0 and 1)?
____ a. It gets stretched horizontally
____ b. It gets compressed vertically
____ c. It shifts horizontally
____ d. It gets reflected along the x-axis

**Q6:** What happens to the function when you make ***g*** negative?

____ a. It gets stretched vertically
____ b. It gets compressed horizontally
____ c. It shifts horizontally
____ d. It gets reflected along the x-axis

- Click on the "+" sign on the top left to get back to normal view

**Q7:** Try to fit the data as closely as possible by manipulating ***g***. What is a good value, or range of values for ***g***?

## 2) Group Discussion:

- According to the model, acceleration due to gravity, ***g*** is somewhere between 9.1 $m/s^2$ to 9.5 $m/s^2$. Really? The *minimum* value of ***g*** is around 9.78$m/s^2$ at the equator.

- Try to set the value of ***g*** at 9.8 $m/s^2$. Notice what happens. Do you get a good fit?

- Why might be this so? Discuss with your classmates and write your conclusion here.

**3) Continue to Investigate**: How does the speed of a roller coaster depend on its height?

**Apply your model:**

**Q8:** According to your model, what speed is a 100m high roller coaster expected to attain? Choose the most likely answer.

____ a. 19 m/s
____ b. 43 m/s
____ c. 55 m/s

**Q9:** Based on your data, how big a drop does a roller coaster need to go 90 meters per second? Choose the most likely answer.

____ a. 445 m
____ b. 725 m
____ c. 865 m

**Q10:** If a roller coaster loses a lot of energy to friction as it converts potential to kinetic energy, where will its point appear on the graph?

____ a. Above the curve
____ b. Below the curve
____ c. Right on the curve

**Challenge!** Explain how you determined the position of the roller coaster on the graph based on the original energy formulas.

| ★ *Tuva Tip: Hint* ★ |
|---|
| ➔ Consider how speed is affected when energy is lost to friction. |

## 5) Challenge Problem:

The orthodox value for $g$ is 9.81 $m/s^2$.

Use the information above and the data you have used to estimate what percent of the energy a typical roller coaster loses to friction as it converts potential to kinetic energy on that big hill.

| ★ *Tuva Tip: Hint* ★ |
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| ➔ Think about what would happen if there were no loss of energy due to friction |

## 5) Group Discussion:

- The Beast, in Kings Mills, seems to have more kinetic energy at the bottom than it had potential energy at the top. How could that be?

★ *Tuva Tip: Hint* ★

➔ Consider if these roller coasters go over the top with some speed

➔ Consider what mechanism was possibly used to pull them to the top?

➔ At the top did they completely stop?