Tuva Activity: Relationship between Speed and Height of Roller Coasters

Activity description:

This activity involves a bit of physics. Students do not need to be experts but a basic knowledge of conservation of energy helps in understanding the form of the function.

The central question in the activity is: *How does the top speed of a roller coaster depend on its height?*

In this activity, students will explore and create a square root function. Students fit a function to a scatterplot of the data to investigate the question. They manipulate a variable parameter to fit the data as well as possible and find the largest and smallest values of the parameter for which the function still looks reasonable.

Tuva Teaching Notes:

➔ *The mechanics of fitting non-linear data in Tuva are different from fitting linear data. But the goal is the same— to find a function that fits the data as closely as possible, interpret the meaning of the parameters, and to find the function's logical connection to the data.*

➔ *In case of non-linear data, students will face the additional challenge of guessing the form of the function. Sometimes the shape would be obvious enough for them to judge the form. But there will be many instances when the context will play a crucial role in judging the form. It is recommended that some time be spent on discussing these aspects.*

➔ *We also offer a print-version of this activity for students, accessible on our website.*

Introduction

Tuva dataset: **Roller Coasters** https://tuva.la/2LL7gMR

Background about the data:

These data are about 20 roller coasters. They contain information about the materials they're made of, how fast they go, how high they are.

Learning Objectives:

Students will be able to:

- Use scatterplots to investigate the relationship between top speed and height of roller coasters
- Construct a function to model the non-linear relationship as an equation
- Interpret the parameter of the function in context of the data

Tuva Teaching Notes:

➔ *Students need to have a basic understanding of conservation of energy.*

➔ *Familiarity with square root function and its graph will be of additional help in doing this activity.*

Question 1:

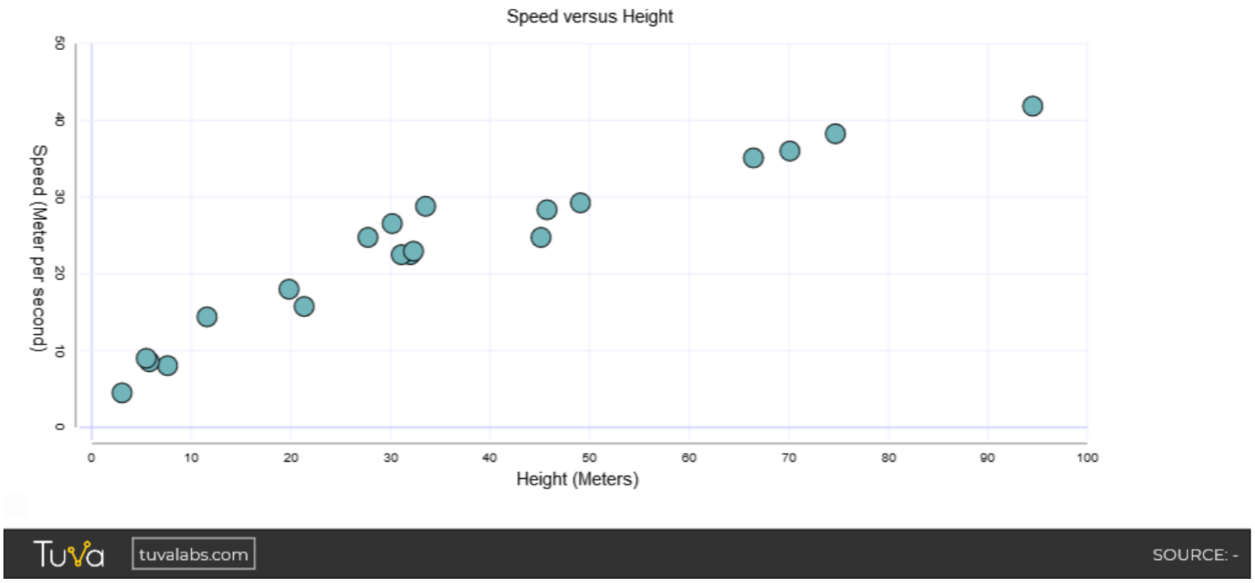
In this multiple-choice item, students are asked to identify the independent variable in the data. (Answer: B. Height)

Question 2:

Students are further asked to choose the statement explaining the reason why height is the independent variable. (Answer: A. Height causes speed, so height is the independent variable)

Students are asked to put the appropriate attributes on the axes by dragging *height* to the x-axis and *top speed* to the y-axis.

Their graph should look like this:



Graph details: The relationship between height and speed is direct in nature as evident from the positive slope. When height increases, speed also increases. The relationship is also non-linear. The positive slope is not constant through the graph, it decreases.

Tuva Teaching Notes:

- ➔ *Students are expected to be fluent in distinguishing between linear and non-linear relationships at this stage. Regardless, there could be a few who still feel confused.*
- ➔ *Have them use the Tuva movable line to measure the slope at different intervals of the graph and deduce that the decreasing slope indicates a non-linear relationship.*

Students are guided to apply the law of conservation of energy to find the function that models the relationship between speed and height.

Tuva Teaching Notes:

- ➔ *Have students draw from their own experiences with roller coasters to garner interest.*

- *It is possible that a few students think that roller coasters are driven by an engine. Take time to discuss how they are entirely gravity-driven.*
- *Allot time for a quick brush-up on basic concepts such as kinetic energy, potential energy, position, velocity and acceleration in the context of roller coasters.*

Question 3:

Students are given a multiple choice question where they are asked to solve the energy equations for speed (*v*) (Answer: C. v = $\sqrt{2gh}$).

Tuva Teaching Notes:

- *Students need a square root function to approximate the relationship.*

  Thus, $y = \sqrt{2gx}$

  where g is a parameter.

- *A parameter is a constant of the situation that may be unknown. In this case, we know it's the acceleration due to gravity (g).*
- *Because there is no direct way to enter radicals into the function editor, students will need to select the function:* a*√x *(the generic form of the square root function) from the drop-down menu. From here, they will modify the equation to fit the context.*
- *Students need to delete the parameter "a" because it is not needed. They can then continue to type in the equation.*
- *The Tuva function editor lets you use x and y instead of actual attribute names. When students type in the function, they need to input* $\sqrt{2gx}$ *instead of* $y = \sqrt{2g * height}$.
- *Students may look at the graph and say, "this looks linear!" Direct their attention to two things:*

- *We have reasons from physics to think that* $y = \sqrt{2gx}$
- *The graph ought to go through the origin if it were a linear relationship. Does it in this case? Have them use the Tuva Least Squares Line to fit the data.*

➔ *Another way to look at this is to ask, what speed should you get with a drop in height of zero? The line predicts about 9 m/s, which doesn't make sense.*

Students are instructed to input the function in the function editor. Their attention is drawn to the function that appears on the graph as well as the parameter g that appears as a slider below the function.

Question 4:

Students are asked to choose the statement that describes the behaviour of the function as they make g larger than 1. (Answer: B. It gets compressed horizontally.)

Question 5:

Students are asked to choose the statement that describes the behaviour of the function as they make g smaller but keep it positive. (Answer: A. It gets stretched horizontally.)

Question 6:

Students are asked to choose the statement that describes the behaviour of the function as they make g negative. (Answer: D. It gets reflected along the y-axis.)

Tuva Teaching Notes:

➔ *Students vary the parameter themselves (using a slider) and can see the function move in relation to the data.*

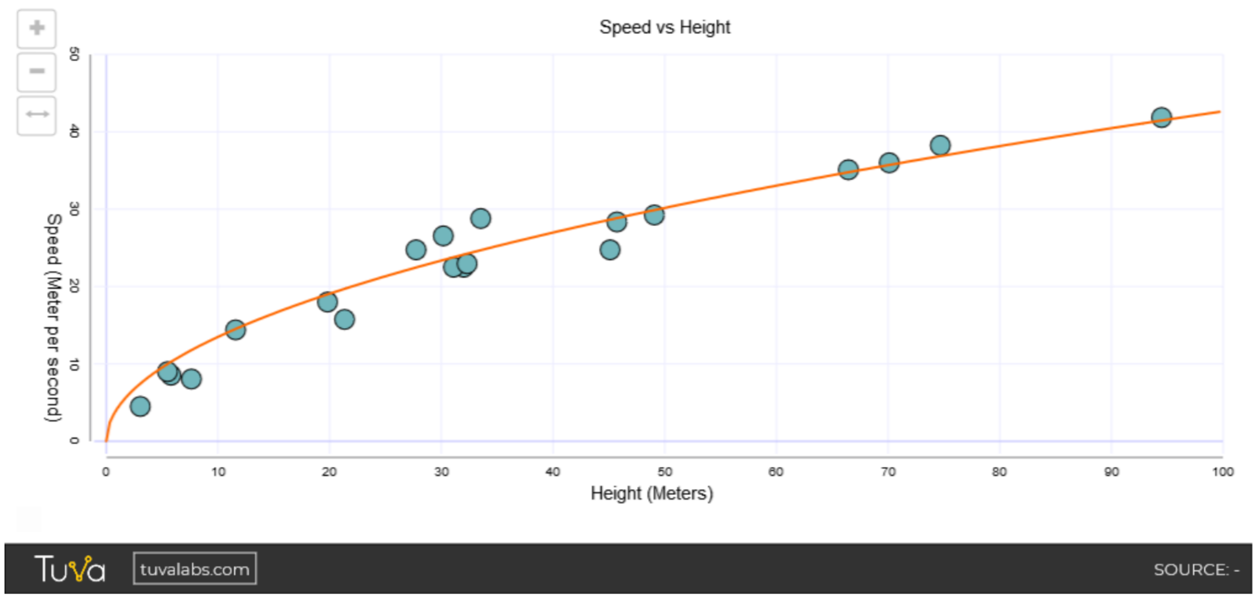
➔ *Allow them to play with the slider and ask them to choose a point on the function and hover over it. The coordinates at that point will show in a tooltip.*

> ➔ *Once they increase or decrease g, ask them to go back to approximately the same point on the function (they can use a reference line on x) and see how the coordinates have changed.*

Question 7:

Students are asked to manipulate *g* until they think they have a good enough fit and report the value. They are also asked to report the value of *g* as a range. (Answer: Answers will vary but a range could be between 9.0 and 9.5 m/s².)

The fitted graph will look something like this:



Group Discussion: Students discuss as a group why the model gives a lower value for *g* than the standard value (9.81 m/s²) they are used to. They are instructed to input the standard value of *g* to see if they get a good fit.

> **Tuva Teaching Notes:**
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> ➔ *When the roller coaster hurtles down, it loses some of its energy due to friction, thus gradually slowing it down. Therefore, we get a much smaller value for g.*

- ➔ *Note that in this context, students get the function up front. They don't really have to figure out what the parameter g means.*
- ➔ *Regardless, they can still use the data to reason about the situation. They can make predictions and even draw conclusions about whether the function really works.*

Question 8:

Students apply their model to choose the most likely speed for a roller coaster which is 100 m high. (Answer: B. 43 m/s)

Question 9:

Students use their model to choose the most likely height of a roller coaster going at 90 meters per second. (Answer: A. 445 m.)

Question 10:

Students are asked to predict where will the point of a roller coaster which loses a lot of energy to friction, appear on the graph. (Answer: B. Below the curve.)

Additional challenge for Question 10:

Students are asked to explain their reasoning behind the choice they make in question 10 based on the original energy formulas. (Answer: Potential energy at the top is converted to kinetic energy as the roller coaster comes down. Since some of the potential energy is converted to heat energy due to friction, the coaster decelerates. This implies that higher the loss to friction, lower the kinetic energy. We know that kinetic energy is equal to $1/2mv^2$. Since mass of the coaster does not change, the speed (v) decreases. For such roller coasters, speed will be lower than what expected from the model. Thus, the their points will appear below the curve.)

Challenge Problem:

Students are asked to use the orthodox value of *g* and the data to estimate what percent of the energy a typical roller coaster loses to friction as it converts potential to kinetic energy on that big hill. (Answer: Approximately 7.24% of energy loss.)

Group Discussion: Students discuss as a group why a few roller coasters seem to have more kinetic energy at the bottom than potential energy at the top.

Tuva Teaching Notes:

- *Reiterate the law of conservation of energy*
- *Guide the groups to acknowledge that there is more energy somewhere. In particular, this means there was some kinetic energy at the top.*
- *Go over the likely assumption that the coaster is always at rest at the top. Ask:*
  - *What if it goes over the top with some speed?*
  - *What mechanism was possibly used to pull it to the top? (Chain, motor...)*
  - *At the top was the coaster COMPLETELY stopped?*
- *The fact is that the roller coaster had some speed at the top and therefore some kinetic energy. Thus, the kinetic energy at the bottom can be as much as that energy PLUS the potential energy due to the fall.*