Tuva Activity: Paragraphs: Relationship between Height and Width

Activity description:

The paragraphs activity is a gentle initiation into the world of nonlinear modeling. In this activity, students will explore and create an inverse relationship.

The idea is fairly simple: If you narrow the margins, a paragraph will get taller. Most students are familiar with this phenomenon working in a word processor.

The central question in the activity is: *How does the height depend on the width?*

Students fit a function to a scatterplot of the data to investigate the question. They manipulate a variable parameter to fit the data as well as possible and find the largest and smallest values of the parameter for which the function still looks reasonable.

Tuva Teaching Notes:

- ➔ *The mechanics of fitting non-linear data in Tuva are different from fitting linear data. But the goal is the same— to find a function that fits the data as closely as possible, interpret the meaning of the parameters, and to find the function's logical connection to the data.*
- ➔ *In case of non-linear data, students will face the additional challenge of guessing the form of the function. Sometimes the shape would be obvious enough for them to judge the form. But there will be many instances when the context will play a crucial role in judging the form. It is recommended that some time be spent on discussing these aspects.*
- ➔ *We also offer a print-version of this activity for students, accessible on our website.*

Introduction:

Tuva dataset: **Paragraphs: Height and Width** https://tuva.la/2KNQEI0

Background about the data:

These data have been collected by measuring the height and width of 7 paragraphs. All the paragraphs have the same text and font size, only the width has been varied by changing the margins. If you want the students to collect their own data, have them measure paragraphs in a word processor, using the rulers it provides. We also offer these paragraphs in a printable handout which students can use for measurements.

Learning Objectives:

Students will be able to:

- Use scatterplots to investigate the relationship between height and width of paragraphs
- Construct a function to model the non-linear relationship as an equation
- Interpret the parameter of the function in context of the data

Tuva Teaching Notes:

- ➔ *Students need to have a basic understanding of area and proportion.*
- ➔ *Familiarity with inverse proportion and its graph will be of additional help in doing this activity.*

Make a Prediction!

In this open-ended response, students are encouraged to predict what the height versus width graph will look like and explain their reasoning (Answer: The emphasis is not on the correct answer).

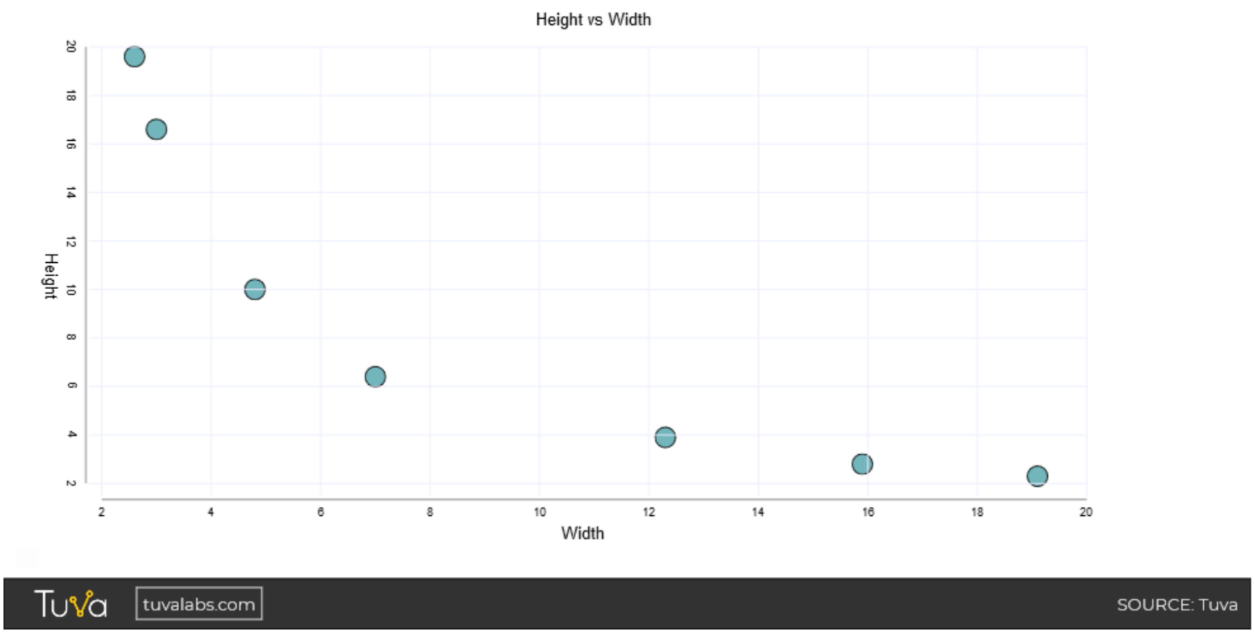
Tuva Teaching Notes:

- ➔ *It is recommended that students have some familiarity with the context to base their prediction on. A quick demo in a word processor should suffice. If you have the time, then have them play with a piece of text themselves.*
- ➔ *Prediction gives them the opportunity to think about the concept ahead of attempting the activity.*
- ➔ *Encourage students to look back at their predictions positively.*
- ➔ *Students' predictions give you a chance to identify potential problems/misconceptions and take steps to address those during the activity.*

Now Graph it:

Students are asked to put the appropriate attributes on the axes by dragging *width* to the x-axis and *height* to the y-axis.

Their graph should look like this:



Graph details: The relationship between height and width is inverse in nature as evident from the negative slope. When width increases, height decreases. The relationship is also non-linear. The negative slope is not constant through the graph, it decreases. This shows that the rate at which height decreases also decreases.

Tuva Teaching Notes:

- ➔ *Students are expected to be fluent in distinguishing between linear and non-linear relationships at this stage. Regardless, there could be a few who still feel confused.*
- ➔ *Have them use the Tuva movable line to measure the slope at different intervals of the graph and deduce that the decreasing slope indicates a non-linear relationship.*

Question 1:

Students are asked to study the shape of the graph to determine how height changes with width and answer a multiple choice question. (Answer: B. Height decreases as width increases.)

Question 2:

They are further asked to choose the statement which correctly describes how the slope of the graph changes. (Answer: A. It becomes less negative.)

Question 3:

Students are asked to choose the option which describes the relationship between height and width correctly. (Answer: D. Negative, Non-linear.)

Group Discussion:

Students work in groups to gauge the form of the function that could model the relationship between height and width. They are prompted to consider the context—what remains constant in the paragraphs and how height changes with width. (Answer: Height is inversely proportional to width. The area of the paragraph remains more or less constant. Hence, the function that would fit the data is height = k/width.)

Tuva Teaching Notes:

- *Students need a function that expresses an inverse proportion. The context plays a crucial role in determining its form. The hyperbola doesn't just fit the points: it arises naturally from the situation. After all, each paragraph has the same text. Therefore the letters take up the same area.*

  Thus, Height = k/width

  where k is a parameter.

- *A parameter is a constant of the situation that may be unknown. In this case, it's the area of the text. An attribute takes on different values depending on which case you're looking at. Sliders let you change parameters and see the effect of a change immediately.*

- *The following questions could be used as discussion prompts:*
  - *What are the variables in the situation?*
  - *What is unknown but constant in the situation?*
  - *What part of your function can you tweak to arrive at a model that fits the data?*

Students are instructed to input the function y = k/x in the function editor. Their attention is drawn to the function that appears on the graph and that parameter *k* appears as a slider below the function. They are also given tips on how to change the value of the parameter

Question 4:

Students are asked to describe the behaviour of the function as they make *k* larger. (Answer: Stretches the function vertically by a scale factor of *k*.)

Question 5:

Students are asked to describe the behaviour of the function as they make *k* smaller but keep it positive. (Answer: Squishes the function vertically by a scale factor of *k*.)

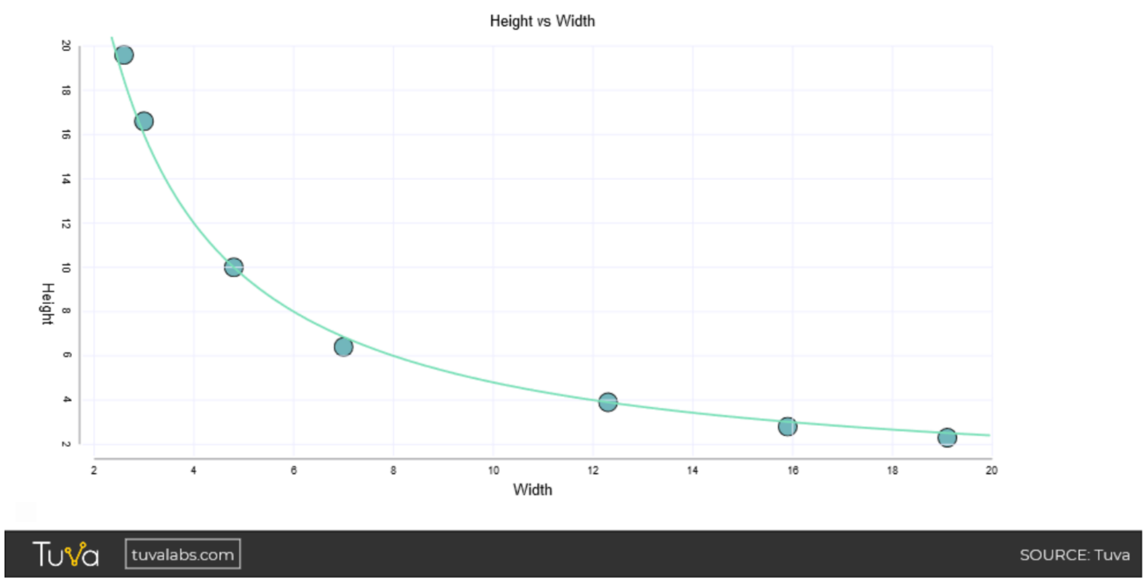
Tuva Teaching Notes:

- *The Tuva function editor lets you use x and y instead of actual attribute names. When students type in the function, they need to input k/x instead of k/width.*
- *Students vary the parameter themselves (using a slider) and can see the function move in relation to the data. Allow them to play with the slider and ask them to choose a point on the function and hover over it. The coordinates at that point will show in a tooltip.*
- *Once they increase or decrease **k**, ask them to go back to approximately the same point on the function and see how the coordinates have changed.*
- *Direct their attention to the fact that the function stretches or squishes by a scale factor of **k**.*

Question 6:

Students are asked to manipulate *k* until they think they have a good enough fit, and then report the value. They are also asked to report the value of *k* as a range. (Answer: Answers will vary but a value of 48 would be a good enough estimate. The range could be between 47 and 50.)

The fitted graph will look something like this:



Tuva Teaching Notes:

- *Students simply vary k until the curve matches the points. This not only gives them their fit, it also helps them understand how changing a parameter value changes a function.*

- *Students will often have to find the value of some parameter that makes a function (a model) work well for a set of data. Here, they are trying to find a good value for k. But they will never know the true value of k. They may find a best value, but only according to some rule (such as least squares).*

- *One strategy is to wiggle k to see what range of values is plausible. That way students can say (for example) that they think k is between 59 and 61. This is* *more flexible than just using significant figures. And it's related to the statistics idea of a confidence interval.*

Questions 7-10:

Students are lead through a series of questions to arrive at the meaning of *k*.

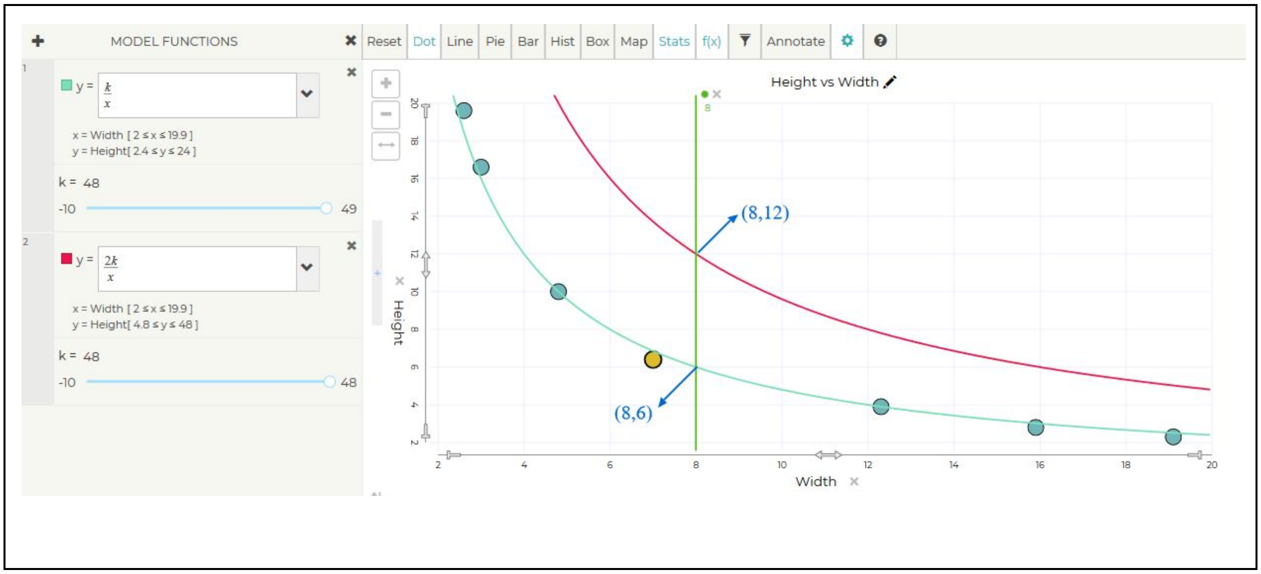
Question 7:

Students are asked to think about how the *relationship* between height and width would be affected if the amount of text in the paragraph is doubled. (Answer: C. The relationship will remain inversely proportional. Doubling the total amount of text roughly doubles the area of the text block. *k* is doubled hence the function stretches by a scale factor of 2. But the underlying relationship remains *unchanged*.)

Tuva Teaching Notes:

- *Encourage students to think about it algebraically. k is the product of height and width (area of the block of text). When k is doubled, how do you balance the equation? You multiply xy by 2, as well. When you rearrange the equation, y is still equal to k/x*

- *Have them create another function $y = 2k/x$. Use the Reference line on X to choose collinear points on the reference line that lie on the two functions. Ask the students to study how the coordinates of the point on the first function change in the second function after k is doubled.*



Question 8:

Students are further asked to envisage what happens to k when the amount of text in the paragraphs is doubled. (Answer: B. k will double. k is the area of the block of text. Doubling the total amount of text, results in the area being roughly doubled, too.)

Question 9:

Continuing the meaning making exercise, students are asked to figure out the units for k (Answer: B. cm^2).

Question 10:

Students are asked to interpret the meaning of the parameter k in their function in context. (Answer: k is the area of the paragraphs.)

Group Discussion:

Students discuss as a group why all the points do not lie exactly on the line. Further, they deliberate about what happens to their model when x is equal to zero and if that makes sense in the context of the paragraphs.

Tuva Teaching Notes:

- ➔ *The amount of text in each line varies because of word wrap. This introduces some variability in the data.*
- ➔ *As the width approaches 0, according to the formula, the height approaches infinity. That makes sense in the context: as the width of the paragraph gets smaller, the letters will all have to pile up and make the paragraph really, really tall. Similarly, as the width* $\rightarrow \infty$*, the height* $\rightarrow 0$*.*
- ➔ *This is a good opportunity to reinforce that the model is not the reality. It is an estimation of what might be, and to make this generalization we often ignore the variability.*