Tuva Activity: Relationship Between Height and Font Size

Activity description:

In the paragraphs activity, students looked at the same text set in columns with different widths. They saw that the height of the paragraphs depended on the width: wide columns made short blocks of text; narrow ones made tall columns. In this activity, they will do something similar. But here, the width stays constant while the text is set in different font sizes.

The central question in the activity is: *How does the length of the paragraph depend on the size of the font?*

Students fit a quadratic function to a scatterplot of the data to investigate the question. They manipulate a variable parameter to fit the data as well as possible and find the largest and smallest values of the parameter for which the function still looks reasonable.

Tuva Teaching Notes:

- ➔ *The mechanics of fitting non-linear data in Tuva are different from fitting linear data. But the goal is the same— to find a function that fits the data as closely as possible, interpret the meaning of the parameters, and to find the function's logical connection to the data.*
- ➔ *In case of non-linear data, students will face the additional challenge of guessing the form of the function. Sometimes the shape would be obvious enough for them to judge the form. But there will be many instances when the context will play a crucial role in judging the form. It is recommended that some time be spent on discussing these aspects.*
- ➔ *We also offer a print-version of this activity for students, accessible on our website.*

Introduction

Tuva dataset: **Paragraphs: Font Size** tuva.la/2L2BWQT

Background about the data:

These data have been collected by measuring the length and font sizes of 10 paragraphs. All the paragraphs have the same text and width, only the font size has been varied. If you want the students to collect their own data, have them choose different font sizes and record the corresponding length of the column of text in a word processor, using the rulers it provides. We also offer these paragraphs in a printable handout which students can use for measurements.

Learning Objectives:

Students will be able to:

- Use scatterplots to investigate the relationship between font size and length of paragraphs
- Construct a function to model the non-linear relationship as an equation
- Interpret the parameter of the function in context of the data

Tuva Teaching Notes:

➔ *Familiarity with quadratic function and its graph will be of additional help in doing this activity.*

Think Ahead/Make a Prediction:

In these open-ended questions, students are encouraged to predict what the height versus font size graph will look like and explain their reasoning. (Answer: The emphasis is not on the correct answer.)

Tuva Teaching Notes:

➔ *It is recommended that students have some familiarity with the context to base their prediction on. A quick demo in a word processor should suffice. If you have the time, then have them play with a piece of text themselves.*

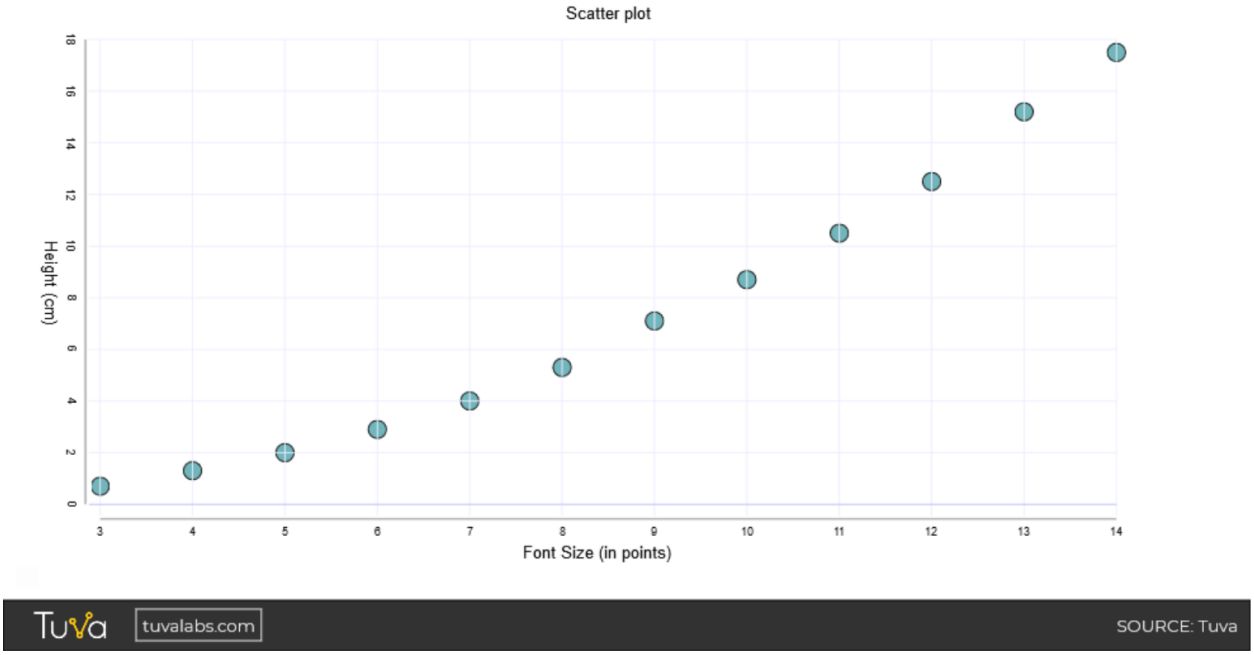
➔ *Prediction gives them the opportunity to think about the concept ahead of attempting the activity.*

➔ *Encourage students to look back at their predictions positively.*

> ➔ *Students' predictions give you a chance to identify potential problems/misconceptions and take steps to address those during the activity.*

Students are asked to put the appropriate attributes on the axes by dragging *font size* to the x-axis and *height* to the y-axis.

Their graph should look like this:



Graph details: When font size increases, height also increases. The relationship is also non-linear. The positive slope is not constant through the graph, it increases. This shows that the rate at which height increases also increases.

Question 1:

Students are asked to study the shape of the graph to determine how height changes with font size (Answer: B. Height increases as font size increases).

Group Discussion:

Students work in groups to make convincing arguments as to why the relationship is non-linear. (Answer: Students are expected to employ informal means such as looking

up the coordinates of a few points to find out how height changes in relation to font size over different intervals and deduce that since the slope is not constant through the graph, it is non-linear. The emphasis is not on the correct answer at this stage.)

Tuva Teaching Notes:

- ➔ *Students are expected to be fluent in distinguishing between linear and non-linear relationships at this stage. Regardless, there could be a few who still feel confused. Additionally, the relationship may not seem non-linear judging by eye.*

- ➔ *Have them use the Tuva movable line to measure the slope at different intervals of the graph and deduce that the increasing slope indicates a non-linear relationship.*

Questions 2-4

Students work through questions 2-4 to gauge the form of the function that could model the relationship between height and font size.

Question 2:

Students are asked to find the height of a paragraph set in 3-point font from the graph (Answer: B. 0.7 cm).

Question 3:

Students are asked to find out the approximate increase in height when font size is doubled from 3-point to 6-point (Answer: C. Height quadruples).

Question 4:

Students are asked to judge the form of the function that best approximates the relationship using the pattern they saw earlier (Answer: C. Quadratic).

Tuva Teaching Notes:

- ➔ *Students need a quadratic function to approximate the relationship between height and font size. 14-point characters are bigger than 7-point characters,*

so the 14-point paragraph will have a greater area. How much greater? A 14-point "M" is twice as high as a 7-point "M."

➔ *Note that "M" is twice as tall—and twice as wide. Which means that the characters in the 14-point font take up four times the area as the ones in the 7.*

➔ *So when you double the font, you quadruple the area. And that quadruples the height here. So the height should be the font size squared.*

Group Discussion:

Students discuss to articulate why the function is quadratic. (Answer: Students are expected to argue that in a quadratic function, if you double x, y increases by a factor of 4? In this situation, height of the paragraph roughly quadruples when we double the font size. Hence, the function is quadratic.)

Students are instructed to input the function in the function editor. Their attention is drawn to the function that appears on the graph as well as the parameter *k* that appears as a slider below the function.

Question 5:

Students are asked to describe the behaviour of the function as they make *a* larger than 1. (Answer: A. Stretches the function vertically.)

Question 6:

Students are asked to describe the behaviour of the function as they make *a* smaller but keep it positive. (Answer: B. Compresses the function vertically.)

Question 7:

Students are asked to describe the behaviour of the function as they make *a* negative. (Answer: D. Reflects the function along the x-axis.)

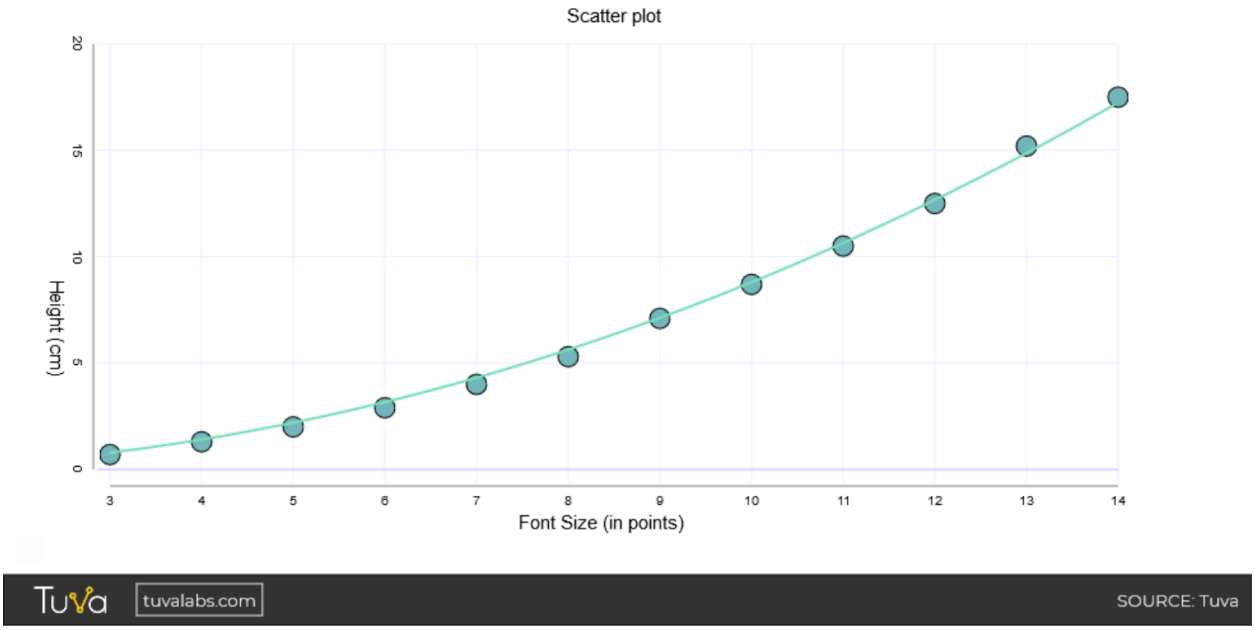
Tuva Teaching Notes:

- ➔ *The Tuva function editor lets you use x and y instead of actual attribute names. When students type in the function, they need to input a * x^2 instead of a * (font size)^2.*
- ➔ *Students vary the parameter themselves (using a slider) and can see the function move in relation to the data. Allow them to play with the slider and ask them to choose a point on the function and hover over it. The coordinates at that point will show in a tooltip.*
- ➔ *Direct their attention to the fact that the function stretches or squishes by a scale factor of a.*

Question 8:

Students are asked to manipulate *a* until they think they have a good enough fit, and report the value. They are also asked to report the value of *a* as a range. (Answer: Answers will vary but a value of 0.087 would be a good estimate. The range could be between 0.085 and 0.089.)

The fitted graph will look something like this:



Tuva Teaching Notes:

- ➔ *Students simply vary **a** until the curve matches the points. This not only gives them their fit, it also helps them understand how changing a parameter value changes a function.*

- ➔ *Students will often have to find the value of some parameter that makes a function (a model) work well for a set of data. Here, they are trying to find a good value for **a**. But they will never know the true value of **a**. They may find a best value, but only according to some rule (such as least squares).*

- ➔ *One strategy is to wiggle a to see what range of values is plausible. That way students can say (for example) that they think a is between 0.085 and 0.089. This is more flexible than just using significant figures, and is related to the statistics idea of a confidence interval.*

Question 9:

Students are asked to predict the height of a paragraph set in 18-point font size. They are further asked to explain why there answer makes sense. (Answer: Answers may vary but anything between 27.5 cm to 28.8 cm should be acceptable. Students are expected to reason that a 9-point paragraph is 7.1 cm tall. When font size is doubled to 18, height should quadruple. 7.1*4 = 28.4 cm. Hence, the answer makes sense.)

Tuva Teaching Notes:

- ➔ This is a good opportunity to reinforce that the model is not the reality. It is an estimation of what might be, and to make this generalization we often ignore the variability.

Question 10:

Students work in groups to figure out the font size of a paragraph as tall as LeBron James (Answer: 48-points).

Challenge Problem:

Students figure out the way to make a 10-page text set in 10-point font fill up 20 pages (Answer: Change the font size to approximately 17-points).